Estimation of the complete stress tensor to 8 km depth in the KTB scientific drill holes: Implications for crustal strength

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Abstract. For many years, in situ stress in the brittle crust has been measured at relatively shallow depth and related to the mechanical behavior of the crust as inferred from laboratory studies and faulting theory. A continuous profile of the magnitudes and orientations of the three principal stresses has been estimated to depths of 7.7 km and 8.6 km in the German Continental Deep Drilling Program (KTB). This was achieved by hydraulic fracturing tests at relatively shallow depth (1-3 km), estimates of the magnitude of the least horizontal principal stress provided by modified hydraulic fracturing experiments at 6 km and 9 km depths, and analysis of compressional (breakouts) and tensile (drilling-induced tensile wall fractures) failures of the borehole wall over nearly the entire depth of the KTB borehole. The orientation of the maximum horizontal principal stress was found to be uniform with depth with an orientation of N160°±10°E, which is consistent with the average orientation found throughout western Europe. The only significant change in stress orientation was observed directly below a major fault zone crossing the borehole. The profile of stress magnitudes we have obtained demonstrates that to a depth of 8 km, the state of stress in the brittle crust in southern Germany is in frictional equilibrium. That is, the ratio of shear to normal stress as resolved on preexisting faults which are well-oriented to the in situ stress field is comparable to their frictional strength based on predictions of Coulomb faulting theory for a coefficient of friction of about 0.7 and near-hydrostatic pore pressure.

Introduction

The KTB site (Kontinentales Tiefbohrprogramm der Bundesrepublik Deutschland) (Figure 1) is located in the Oberpfalz region in northeastern Bavaria at the western end of the Bohemian massif. A repeated sequence of metamorphic rocks, mainly highly folded and steeply dipping gneisses and massive amphibolites, were encountered at depth. For a detailed description of the geology and tectonic setting at the KTB site, see Hirschmann [1993] and Emmemann and Lauterjung [this issue]. Two deep boreholes were drilled during the KTB project, a continuously cored "pilot hole" reaching a final depth of 4 km in 1989, and the "main hole" with a final depth of 9.1 km reached at the end of 1994. The stress orientations displayed in the map (Figure 1) are part of the World stress map (WSM) database and are determined by various methods, such as earthquake focal plane solution, overcoring, hydraulic fracturing, and analysis of geological stress indicators [Zoback, 1992; Müller et al., 1992]. They indicate the regional NW-SE orientation of the greatest horizontal principal stress throughout central Europe.

Numerous investigators have introduced theoretical strength profiles for the crust, which basically separate the crust in an upper brittle and lower ductile regime. The strength in the brittle part is controlled by the frictional strength of preexisting, favorably oriented faults, and varies with the tectonic regime [Sibson, 1974], whereas the strength of the lower ductile part is described by flow laws for appropriate rock types, temperatures, and strain rates [e.g., Götze and Evans, 1979]. The transition between these zones is usually depicted as the intersection of the two strength envelopes, but it is obvious [e.g., Scholz, 1990] that the transition occurs over an interval of several kilometers as the rheology of the material gradually changes from brittle to ductile. Commonly, the 300°C isotherm is thought to be the onset of plasticity in rocks of gneissic and granitic composition. As the temperature gradient measured at the KTB site is 27.4°C/km [Clauser et al., this issue], this temperature corresponds to a depth of ~11 km.

Theoretical stress profiles are based on laboratory investigations of rock friction and rock rheology at high temperatures and high pressures, and the assumption that the crust is in a state of failure equilibrium [e.g., Kohlstedt et al., 1995]. It is important to try to verify the validity of both the applicability of laboratory derived strength parameters to the crust, and the assumption that the crust is in a state of failure equilibrium. A first compilation of in situ stress magnitude measurements was made by McGarr and Gay [1978] and McGarr [1980] and was compared to the stresses predicted with the assumption that the state of stress is in frictional equilibrium on preexisting faults with laboratory derived coefficients of friction of 0.6-1.0 [Byerlee, 1979, Brace and Kohlstedt, 1980]. They found that strain relief stress measurements from mines in South Africa and Canada, and limited hydraulic fracturing results from the United States, fit within the bounds given by Byerlee's law, taking into account the appropriate pore pressure for the area of investigation. Zoback and Healy [1984] showed that available stress...
measurements in intraplate regions of active faulting were, in fact, consistent with estimates based on Coulomb faulting theory and laboratory-derived coefficients of friction to a depth of about 3 km.

In fact, essentially all in situ stress magnitude data available in intraplate regions (United States, South Africa [McGarr and Gay, 1978]; Scandinavia, central Europe [Rummel, 1986; Rummel et al., 1986]; Monticello, South Carolina [Zoback and Hickman, 1982]; Cornwall, England [Pine et al., 1983]; Yucca Mountain, Nevada [Stock et al., 1985]; Fenton Hill, New Mexico [Barton et al., 1988]; Cajon Pass [Zoback and Healy, 1992]) are consistent with the concept that stress magnitudes in the crust are in equilibrium with the frictional strength of the crust, and that laboratory derived coefficients of friction \( \mu \) in the range of 0.6 to 1.0 [Byerlee, 1978] can be applied to faults in situ. As these measurements are essentially all shallower than 3 km, an outstanding question about crustal stress magnitudes is whether the findings at relatively shallow depth extend to midcrustal depths approaching the brittle-ductile transition.

**Integrated Stress Measurement Strategy**

As the KTB project evolved, we undertook development of a strategy for determining the complete stress tensor (the magnitude and orientation of the three principal stresses) to as great a depth as possible. The continuity of the stress profile along the borehole and its resolution with depth were important aims of the experimental plan as well as the maximum depth of the stress determinations. As commonly used methods for stress determination, like overcoring, hydraulic fracturing, or core-based methods, allow for neither the determination of a continuous profile nor very deep measurements, alternatives...
and modifications had to be developed to reach these ambitious goals.

An "integrated stress measurement strategy" (ISMS) was developed which combines various methods to establish as complete a stress profile as possible. Assuming that the vertical stress is a principal stress, the complete stress tensor can be determined by the orientation and magnitude of the least and maximum horizontal principal stresses, \( S_h \) and \( S_v \) respectively. The magnitude of the vertical stress at depth \( z \) is defined by the load of the overburden \( (S_v = \rho g z) \) calculated for a mean density \( \rho \) of 2800 kg/m\(^3\).

Huenêst et al. [this issue] describe the results of the investigation of the geohydraulic properties of the crust at the KTB site. They were able to establish a profile of the pore pressure to the final depth of 9.1 km of the KTB main hole. A transition from fresh water to saline water at 2000 m depth causes a step in the pressure gradient from 9.8 to 10.5 MPa/km. Below 6000 m depth to the bottom of the hole the gradient increases further and leads to an average gradient of the formation pressure of 11.5 MPa \( \pm \) 0.5 MPa/km for the entire investigated depth to 9.1 km. The pore pressure is essentially hydrostatic (when corrected for the salinity of the fluids) to the final depth of the KTB main hole.

The analysis of the orientation of breakouts and drilling-induced fractures, by means of borehole measurements, is used to determine the orientation of the principal horizontal stresses. To a depth of 3 km it was possible to determine both the magnitude of \( S_h \) and \( S_v \) by classical hydraulic fracturing experiments [Baumgärtner et al., 1990]. Below this depth, a lower bound for the magnitude of \( S_h \) was determined at 6 km and 9 km depth by modified hydraulic fracturing tests necessitated by the hostile conditions encountered at these great depths in the KTB main hole. Technically limiting factors were the high temperature gradient of 27.4°K/km [Zöll, 1993; Clauser et al., 1994, this issue], the high mud and fracture pressures, and the ubiquitous failures of the borehole wall. The modified hydraulic fracturing experiments at 6 km and 9 km depth were not intended to determine the magnitude of \( S_h \), which can be obtained by classical hydraulic fracturing tests, but only the magnitude of \( S_v \). Owing to this lack of information on the magnitude of \( S_v \), from hydraulic fracturing, it was necessary to develop methods to estimate the magnitude of \( S_v \) from the occurrence of breakouts and drilling-induced tensile wall fractures [Brudy, 1995] using the knowledge of the magnitude of \( S_h \) gained from the modified hydraulic fracturing experiments. The combination of the two estimations allows the setting of limits on the ranges of magnitudes for both \( S_h \) and \( S_v \). The ISMS was therefore based on the results of the modified hydraulic fracturing tests and the availability and quality of well bore logging data (four-arm caliper, borehole televiewer (BHTV) [Zemanek et al., 1970] and formation microscanner/formation microimager (FMS/FMI) [Ekstrom et al., 1987]) over the entire depth of the KTB main hole that indicate the occurrence of well bore failures.

In the following sections we first describe the various hydraulic fracturing tests and their results, then the methods to estimate the magnitude of the greatest horizontal principal stress from the occurrence of breakouts and drilling-induced fractures.

Hydraulic Fracturing Tests

Hydraulic fracturing measurements are a critically important part of the determination of the stress in the ISMS. At shallow depth, they were used to determine the magnitude of both horizontal principal stresses (experiments to 3 km in the KTB pilot hole), and at greater depth to establish a lower bound for the magnitude of the least horizontal principal stress \( S_h \) (at 6 and 9 km depth in the KTB main hole). The relatively shallow "classical" hydraulic fracturing measurements were also important as a means of calibrating the ISMS utilized at greater depth in the KTB main hole.

Hydraulic fracturing was first described as a stress measurement technique by Haimson and Fairhurst [1967] and since has been used worldwide with success (see compilations edited by Zoback and Haimson [1983], Stephansson [1987], and Haimson [1989]).

Hydraulic Fracturing Tests in the KTB Pilot Hole

Conventional hydraulic fracturing was used to determine the state of stress in the KTB pilot hole to a depth of 3 km [Baumgärtner et al., 1990]. Fourteen experiments were attempted in the KTB pilot well, of which seven could be used for determining the state of stress. During the other seven experiments, the maximum pressure capacity of the equipment was not sufficient to initiate hydraulic fracture. The results of the successful experiments are displayed in Figure 2. \( S_h \) is determined from the shut-in pressure \( P_w \) which corresponds to the pressure necessary to hold the fracture open against the normal stress acting on the fracture plane. As the fracture plane is assumed to propagate normal to \( S_h \) (due to the minimum energy principle), \( P_w \) is taken to be equal to \( S_h \). The magnitude of \( S_h \) is estimated using a linear elastic approach, which assumes that poroelastic effects are negligible in low porosity (<1% [Rauen et al., 1990]) crystalline rocks like the gneisses.

![Figure 2](attachment:image.png)

Figure 2. The magnitudes of the horizontal principal stresses measured in hydraulic fracturing experiments at the KTB site are displayed as solid and open squares. The vertical stress is calculated as the overburden load \( (S_v = \rho g z) \) for a density \( \rho \) of 2800 kg/m\(^3\), the gravity acceleration \( g \) of 9.81 m/s\(^2\), and the depth \( z \). Error bars incorporating the uncertainties in the pressure determination and in the determination of the hydraulic tensile strength are displayed for the \( S_h \) magnitudes.
and amphibolites at KTB. This allows $S_h$ to be estimated using the breakdown equation introduced by Hubert and Willis [1957] for a material which is impermeable to the fracturing fluid,

$$P_{bd} = 3S_h - S_H + T - P_0,$$

(1)

where $P_{bd}$ is the breakdown pressure, $T$ is the tensile strength of the rock, and $P_0$ is the pore pressure. Baumgärtner et al. [1990] argue that due to low porosity of the gneiss encountered in the pilot well, and due to the probable existence of dense drilling mud which prevents infiltration of fluid in the rock prior to fracture initiation, the pore pressure can be neglected in (1) leading to

$$S_H = 3P_{nl} - P_{bd} + T.$$  

(2)

Schmitt and Zoback [1989] discuss the theoretical reasons why this may be valid. Uncertainties in the determination of the $S_n$ magnitude are caused by (1) the uncertainties in the determination of the pressures $P_n$ and $P_{nl}$ [Baumgärtner et al., 1990], and (2) by the uncertainty involved in the determination of the hydraulic tensile strength $T$ in equation (2), which is determined as the difference between $P_n$ and the reopening pressure $P_{nl}$. For hydraulic fracturing experiments at relatively great depth, the reopening pressure $P_{nl}$ might be slightly overestimated as the change in the hydraulic stiffness of the testing system when the fracture first starts opening at the borehole wall might be too small to be recognised. This overestimation of $T$ leads to too low a $S_n$ magnitude. As the differences between $P_n$ and $P_{nl}$ are of the order of 5 MPa [Baumgärtner et al., 1990], but laboratory values for the hydraulic tensile strength $T$ are $\sim 10$ MPa, the error bars in Figure 2 account for an $\sim 5$ MPa underestimation of the $S_n$ magnitude.

The stress magnitudes displayed in Figure 2 show a thrust faulting regime for the measurements to 800 m depth ($S_h$ is the least principal stress) and a strike-slip stress regime for all measurements below this depth. The orientation of fractures induced by the hydraulic fracturing experiments was detected by postfracturing FMS logging. The mean orientation of the induced fractures (indicating the azimuth of $S_h$) was found to be N149°E±15° [Baumgärtner et al., 1990].

Hydraulic Fracturing Experiment at 6 km Depth in the KTB Main Hole

The hydraulic fracturing experiment at 6 km depth was planned as a modified drill stem test using a casing packer in the 34-cm casing (13 3/8 inch). A 17.5-m open hole section was available below the cemented casing, with 13 m (6018 m to 6031 m) newly drilled hole with a bit size of 31 cm (12 1/4 inch). Owing to safety considerations, the maximum injection pressure was limited to 53.5 MPa [Engeser et al., 1993]. To isolate the test interval, a mechanical packer was set into the casing by loading it with a weight of 15-20 t. The packer used in the experiment was capable of withstanding differential pressures up to 51.7 MPa (7500 psi) at temperatures of 250°C.

The downhole pressure and flow rate versus time during the hydraulic fracturing procedure are presented in Figure 3. The significant decrease of the rate of pressure build up at constant pumping rate is marked by the A. When the pumping rate was increased to 34 L/min (B) the pressure again increased. Unfortunately, the pumping had to be stopped at a pressure of 115.3 MPa, the pressure limit given by safety considerations. Owing to this pressure limitation, it was not possible to test the system at higher flow rates and pressures. In a classical hydraulic fracturing experiment, the maximum pressure should be almost independent of the pumping rate [Lee and Haimson, 1989; Li, 1989], and thus the observed flow rate sensitivity of pressure is not consistent with the interpretation that a classical hydraulic fracture is opened at point A. For this reason, we think that the pressure value marked by A in Figure 3 cannot be

![Figure 3. Downhole pressure and flow rate during the hydraulic fracturing experiment at 6 km depth in the KTB main hole are presented. Marked by A is the sudden decrease in the rate of pressure build up and by B the change to higher flow rate which is connected to a further increase in downhole pressure. Owing to technical limitations of the downhole pressure pumping at the high flow rate had to be stopped at 115 MPa (sudden decrease of flow rate shortly after 2130 LT). At the point marked by "shut-in" the pumping is stopped and the test interval is closed. The interpretation of this test is that no new hydraulic fracture was created during the test. Therefore the highest pressure reached during the test can be interpreted as a lower bound for the magnitude of $S_h$.](image-url)
interpreted as a true "breakdown" pressure. Instead, the maximum pressure of 113.5 MPa reached in this experiment can be assumed to be a lower bound for the magnitude of \( S_\sigma \), as this pressure may have been reached without creating a new fracture, and the true magnitude of \( S_\sigma \) could be greater than this pressure. Nevertheless, as the pressure was not increasing during injection at constant flow rate (between points A and B), this pressure may be quite close to \( S_\sigma \).

Hydraulic Fracturing Experiment at 9 km Depth in the KTB Main Hole

A combined hydraulic fracturing/induced seismicity experiment was carried out in the openhole section of the KTB main hole between 9030 m and 9101 m depth to investigate the hydraulic properties of the rock and to determine the recent tectonic stresses at this depth. The first part of the experiment was a modified hydraulic fracturing test, conceptually very similar to the experiment run at 6 km depth, whereas the second part had the aim to induce seismicity by injection of a relatively large volume of fluid (200 m\(^3\)) into the formation [see Zoback and Harjes, this issue].

From 8625 m to 9101 m the hole was drilled with 16.5-cm bit size (6 1/2 inch). To 9030 m depth, this interval was sealed off with a cemented 14-cm liner (5 1/2 inch), and in the final drilling phase, the hole was deepened to 9101 m. Instead of using a casing packer inside the 14-cm liner (5 1/2 inch), it was decided to use an Internal Seal Assembly (ISA) which was installed on top of the 14-cm liner (5 1/2 inch) at 8625 m. This technique had the advantage that if the cementation was not tight (and hydraulic communication between the open hole section and the top of the liner occurred), the pressure could not reach the annulus above 8625 m. To pressurize the test interval, a perforated "tail pipe" was installed 5 m into the open hole section (for further technical details, see Engesser et al., 1993).

The surface pumping equipment had a range of working conditions from a flow rate of 10 L/min to 560 L/min at a maximum pressure of 70 MPa. To achieve a maximum downhole pressure of 183 MPa, a CaCl\(_2\)/CaBr\(_2\) solution with a density of 1500 kg/m\(^3\) was used as the injection fluid. This fluid was chosen for the experiment as it is free of solid particles and therefore avoids the risk of clogging any permeable fractures during injection of large volumes in the second phase of the experiment.

After a linear pressure increase at the begin of pumping at about 154 MPa, a sharp breakdown of the pressure is observed reducing the pressure to a plateau at about 147 MPa [Zoback and Harjes, this issue]. The second pumping cycle shows first a linear pressure increase with time and then reaches a plateau pressure. The third cycle carried out with the same pumping rate of 50 L/min shows a nonlinear pressure increase and did not reach a constant pressure as expected. While the sudden pressure decrease in the first pumping cycle looks like the typical pressure drop at the initiation of a hydraulic fracture, the later pumping cycles at higher pumping rates were accompanied by higher pressure buildups. At a pumping rate of 560 L/min, the maximum technically feasible surface pressure of 53 MPa was reached, and the flow rate had to be reduced in order not to overcome the pressure limitation. The fact that pressure kept building up during constant-rate injection and that pumping pressure was very sensitive to increased flow rates is contradictory to the interpretation that a hydraulic fracture was initiated during the first pumping cycles. Assuming that no new fracture was initiated at the maximum downhole pressure of 183 MPa establishes this as a lower bound for the magnitude of \( S_\sigma \).

Estimation of \( S_H \) Magnitude From Well Bore Breakouts

Breakouts are zones of failure of the borehole wall in response to high compressive stresses. The stress concentration around the borehole is described by the stress field around a circular opening in an infinite plate of elastic material, as originally given by Kirsch (1988). The analysis of breakouts for this study was done by borehole televiewer (BHTV) measurements. The BHTV is an acoustical device scanning the borehole wall with acoustic pulses emitted during its rotation [Zemanek et al., 1970]. The travel time and the amplitude of the reflected pulses are recorded and, after processing, displayed as images covering 360° of the borehole wall [Barton et al., 1992]. Plate 1 shows the amplitude and the travel time image of a section of the borehole wall in the main well between 4804 m and 4813 m depth. The travel time image is converted to a radius image using the acoustical wave velocity of the drilling mud. Elongations in the amplitude image are clearly seen as bands of purple representing low reflected amplitudes and as yellow and reddish bands in the corresponding radius image representing high travel times. Cross-section examples from four depths are shown in Plate 1. Usually, the breakouts are diametrically opposed at the borehole wall; the wall is rough inside the breakout and smooth outside the breakout. Sometimes breakouts continue over a depth range of several meters and then show minor fluctuations in orientation and width with depth [Shamir and Zoback, 1992; Brudy et al., 1993; Barton and Zoback, 1994]. The breakout section shown here is terminated at a fault crossing the well at 4812.5 m visible as sinusoidal structure (purple to blue) in the amplitude image (Plate 1, left).

The morphology of breakouts is described by three parameters: (1) orientation \( q_o \), (2) opening \( 2F \), and (3) radial depth \( \eta \) [Zoback et al., 1985; Barton et al., 1988]. The breakout opening is the angular section of the borehole affected by compressional failure. The breakout orientation is defined as the orientation of the midpoint of the breakout opening. As the physical relation between the radial depth and the stress field is unclear and also because the radial depth is often hard to determine, we restrict our analysis to breakout orientation and opening angle. Two mechanisms seem to be responsible for the creation of breakouts in the KTB wells: compressive shear failure due to high tangential stresses in the borehole wall, and extensional splitting of thin flakes due to high tangential stresses combined with a lack of "confining pressure" by the drilling mud. The two different mechanisms can be seen on rock samples brought up to the surface in a "junk basket" attached to the drill string above the drill bit (T. Rökel, personal communication, 1993) and from the analysis of cuttings [Lich and Dyuster, 1993]. Despite the fact that the formation of breakouts involves two fracture processes (see also laboratory tests of compressive failure of boreholes which indicate both processes [Haimson and Song, 1993; Lee and Haimson, 1993]) and that the mechanism for stabilization of breakouts is still debated [e.g., Zheng et al., 1989; Cheatham, 1993], it is found that it is possible to relate empirically the breakout opening angle to the far-field principal stresses [Barton et al., 1988; Vernik and Zoback, 1992; Haimson and Song, 1993].
Plate 1. The borehole teviewer emits an acoustic pulse and records amplitude and travel time of the reflected signal. The travel time is converted to radius information with the acoustic velocity of the mud. Displayed are (left) the amplitude and (right) the travel time image of a section of the borehole wall in the main well between 4804 m and 4813 m depth. Breakouts are characterized by bands of purple representing low reflectec amplitudes and by yellow and reddish bands in the corresponding radius image representing high travel times. Four examples of cross sections of the borehole constructed from radius data are shown far right. The displayed cross sections are elongated in the E-W direction.
Figure 4. The occurrence and orientation of breakouts detected in BHTV measurements are shown for the depth interval between 3 km and 6.8 km. Apart from local variations the orientation of the breakouts is constant with depth. Above 6 km depth, sections of stable hole are found, where no breakouts were formed, whereas between 6 km and 6.8 km depth the hole is failing continuously. Gaps in the breakouts indicated in this depth section are due to the fact that the borehole wall was failing so severely that no breakout orientation could be determined.

Figure 4 displays the occurrence and orientation of breakouts with depth in the main well between 3000 m and 6800 m. Apart from some depth sections where no breakouts occurred, for example, around 4700 m, 5000 m, and 5600 m, breakouts are found over the entire investigated depth interval with only very narrow interruptions. Some of these sections are characterized by cataclastically deformed rocks (e.g., 4700 m), which seem to resolve the stresses by shear on preexisting fractures without loosing their strength entirely. In other sections the compressive strength of the intact amphibolite seems to be so high that the strength of the rock is greater than the stress concentration. As this latter case is especially true at shallower depth where stress differences are lower, it would be in general accordance with the observation that in the KTB boreholes breakouts become generally more severe with increasing depth. Apparent gaps in the occurrence of breakouts seen below 6000 m are not, in fact, indications of a stable borehole, rather, just the opposite. These depth sections are so severely enlarged that it is impossible to determine a correct orientation of the breakouts. Apart from local small-scale variations, the breakouts are consistently oriented in ENE-WSW direction along the entire investigated section.

In rocks which are deforming elastically and isotropically to the point of failure (which is generally the case for the amphibolites encountered at KTB [Vernik et al., 1992a]), the stress distribution around an initially undeformed cylindrical borehole is well described by the equations given by Kirsch [1898]. The breakout shape can be described by the area where
the stresses in the rock overcome the in situ compressive strength $C_{\text{et}}$ [Zoback et al., 1985; Vernik and Zoback, 1992]. The contour of the breakout area is the line on which the stress is equal to $C_{\text{et}}$. Following Barton et al. [1988], analysis of the stress state at the point of onset of the breakout on the borehole wall leads to

$$\sigma_{\theta b} (\theta_b) = \left(S_H + S_h \right) - 2(S_H - S_h) \cos 2\theta_b - \Delta P = C_{\text{et}} \text{.}$$

with

$$2\theta_b = \pi - 2\phi_b \text{.}$$

$\theta_b$ is the angle of the onset of the breakout with respect to the orientation of the greatest horizontal principal stress $S_H$. $2\phi_b$ is the breakout opening angle, and $\Delta P$ the difference between the hydrostatic pressure in the borehole $P_{\text{hydr}}$ with pore pressure $P_r$. As the rocks encountered in the KTB boreholes have very low intrinsic porosity and permeability [Rauen et al., 1990; Huenges et al., this issue], the pore pressure $P_r$ is assumed to be negligible for the initiation of breakouts, and thus $\Delta P$ is equal to $P_{\text{hydr}}$ the hydrostatic mud pressure in the borehole. Solving equation (3) for $S_H$ leads to

$$S_H = \frac{C_{\text{et}} + P_{\text{hydr}}}{1 - 2\cos(\pi - 2\phi_b)} - S_h \frac{1 + 2\cos(\pi - 2\phi_b)}{1 - 2\cos(\pi - 2\phi_b)} \text{.}$$

The fluid pressure $P_{\text{hydr}}$ is calculated as the hydrostatic pressure exerted by the mud column, and the magnitude of the minimum horizontal stress $S_h$ is known from hydraulic fracturing experiments. The opening angle of the breakouts is measured from the BHTV data in the main well between 3200 m and 6800 m depth. The mean opening angle for every 50-m interval is displayed in Figure 5a. The standard deviation (shown in Figure 5a as horizontal bars) of the breakout opening angles is due to the variability of the rock strength in a certain depth section. A generalized lithological profile on the right-hand side of Figure 5 is added to show depth sections that were excluded from the analysis because the gneisses in these sections are strongly foliated and have anisotropic strength [Vernik et al., 1992a].

The parameter with the strongest influence on the estimation of the magnitude of $S_h$ is the effective in situ rock strength, $C_{\text{et}}$. In general, the criteria used to describe the failure of rock, such as the Mohr-Coulomb or Griffith criterion, assume that the strength of rock is independent of the magnitude of the intermediate principal stress. However, polyaxial tests of the rock strength [Mogi, 1971] show that intermediate stress can sometimes have a strong influence on rock strength. As the state of stress along a borehole wall is polyaxial (in the $S_3$ direction typically, $s_3 = C_{\text{et}} > s_2 = s_1 > s_1 = s_2$), it is necessary to use a failure criterion which includes the dependence of the rock strength on the intermediate stress. We have chosen the effective shear

![Figure 5](image-url)

**Figure 5.** Breakout opening angle and effective in situ rock strength are important input parameters for the breakout analysis. (a) The mean breakout opening angle for intervals of 50 m length slightly increases with depth. On the right-hand side, depth sections which are drilled in gneisses are marked by grey shading. They are excluded from the stress analysis as their strength is anisotropic. (b) The uniaxial compressive rock strengths measured by Röckel and Natau [1989, 1992] are displayed as solid squares. The dashed lines represent the standard deviation of the uniaxial strength. The in situ effective strength $C_{\text{et}}$, as calculated using the ESEF criterion [Wiebels and Cook, 1968], is shown by the shaded area.
strain energy failure (ESEF) criterion [Wiebols and Cook, 1968] from which rock strength values at polyaxial stress states at the borehole wall can be estimated from triaxial strength measurements [see also Vernik and Zoback, 1992]. The ESEF criterion and its applicability are discussed in more detail in Appendix A.

Uniaxial compressive strength tests were made routinely in the KTB pilot well and later also with core material retrieved from the main well [Röckel and Natou, 1989, 1992]. Triaxial strength tests, which are more reliable for the determination of strength than uniaxial tests, are reported by Vernik et al. [1992a]. As two different sample sizes were used for the uniaxial and triaxial experiments, a comparison of the results is difficult. Röckel and Natou [1989, 1992] used sample diameters of 94 mm and a length to diameter ratio of 1:1. This diameter to length ratio results in overestimated rock strength values [Mogi, 1966] and must be corrected to compare with commonly used length to diameter ratios greater than two. Vernik et al. [1992a] utilized a sample diameter of 25.4 mm and a length to diameter ratio of 2 to 2.5. They found that the amphibolites encountered in the KTB boreholes behaved elastically nearly to the point of failure and had very little strength anisotropy (less than 10%). The KTB gneisses tested had a considerable amount of strength anisotropy, of 42% to 60%. As mentioned above, to avoid problems caused by the highly anisotropic strength of gneisses, we decided to restrict the breakout analysis to depth sections drilled in amphibolite.

It is shown in laboratory experiments that small-diameter holes fail at much higher stresses than large-diameter holes [Haimson, 1990]. This effect is commonly called the scale effect in rock mechanics and has been observed in many triaxial strength tests. In the case of the KTB main borehole, the hole diameter and the size of the breakouts are in the same range as the sample sizes used by Röckel and Natou [1989, 1992]. However, we need to correct their strength values because their samples had a length to diameter ratio 1:1. A correction value of 0.88 was used, based on empirical relations found by Gartung [1979] and International Society of Rock Mechanics [1979]. The corrected measurements are shown in Figure 5b by squares with the mean uniaxial rock strength and the standard deviation are plotted as solid and dashed lines, respectively.

To define the effective in situ strength of the rock Cw, we use the ESEF criterion as described in Appendix A. The values for sliding friction (μs) and for internal friction (μt) (which are necessary input parameters in the ESEF criterion) were found from the triaxial strength tests of amphibolite by Vernik et al. [1992a]. The effective in situ strength under a polyaxial state of stress was determined for the minimum stress being equal to the fluid pressure in the well bore (σf=σt=Pw), and the intermediate stress being equal to the weight of the overburden (σv=σi).

However, the rock strength at which the breakouts actually form is likely to be significantly lower than that determined directly by the ESEF criterion. Vernik and Zoback [1992] and Vernik et al. [1992b] relate this to the dilatancy preceding brittle failure of rocks, which is a common phenomenon. Owing to subcritical microcrack growth parallel to the direction of maximum stress, a slight increase in volume preceding brittle failure is observed. Vernik et al. [1992b] used cylindrical samples from cores retrieved from the KTB pilot well for which the peak strength was previously determined in dry tests. In triaxial tests with a confining pressure of 35 MPa (corresponding to the mud pressure at 3.5 km depth), they loaded the samples to 50% of the peak strength found under dry conditions and then added pressured water (34.5 MPa). Except for one sample (a strongly anisotropic muscovite-biotite gneiss), all samples failed between 3.8 and 72 hours after injection of water. These results imply that subcritical microcrack growth significantly increases the permeability of the rock and allows pore pressure to equilibrate with fluid pressure in the well bore. Thus failure is expected to occur in situ if the circumferential stress exceeds some 50% to 60% of the dry rock strength. Thus, in the analysis of breakouts for the stress magnitude determination, an effective rock strength Cw of 50±5% of the strength C given by the ESEF criterion (Cw=0.5±0.05C) is used. The resulting in situ strengths of the rocks, Cw, are displayed by the shaded area in Figure 5b.

### Estimation of Sw Magnitude From Drilling-Induced Tensile Wall Fractures

Pairs of diametrically opposed drilling-induced tensile wall fractures are found in the KTB holes nearly always parallel to the well bore axis persisting 0.1-1 m along the length of the hole and crosscutting other structures, like foliation and faults (Plate 2). They were first observed in the pilot hole by formation microscanner (FMS) measurements, and later also by BHTV and formation microimager (FMI) logging in the main well. A detailed description of phenomenology and initiation of drilling-induced fractures in KTB is given by [Brudy, 1995], while in the following, only the most important facts necessary for this work are presented. An analysis of the core material

![Image](image-url)
available in the pilot well demonstrated that these fractures are not present in the cores and therefore appear not to be preexisting fractures cut by the well bore [Röckel et al., 1992]. Comparison of the orientation of these fractures to independently determined orientations of the principal horizontal stresses shows that the fractures always occur parallel to the orientation of the maximum horizontal stress $\sigma_h$. Direct comparison of breakouts and drilling-induced fractures in BHTV recordings in the main well shows that drilling-induced fractures always occur 90° off the direction of breakouts [Brudy et al., 1993]. As breakouts are reliable indicators of stress orientation, the constant 90° offset between the breakout orientation and the orientation of the drilling-induced fractures is further evidence that the drilling-induced tensile fractures were initiated by drilling and are themselves reliable indicators of the tectonic stress orientation. Figure 6 displays the occurrence and orientation of drilling-induced fractures in the KTB main hole determined from FMS/FMI measurements. Below 6.2 km depth, drilling-induced fractures are only found around 7 km and 7.7 km depth. Depth sections where the quality of the measurements was not sufficient to detect fractures, owing to the fact that the arms of the FMI tool were not properly attached to the borehole wall (severe enlargements of the borehole diameter), are indicated by ticks on the right-hand side of Figure 6. In these sections it cannot be decided if drilling-induced fractures have been initiated or not. As with the breakouts in Figure 4, the drilling-induced fractures are occurring at a very constant orientation along the entire investigated depth.

In the KTB pilot hole the minimum circumferential stress at the well bore wall induced by tectonic far-field stresses (known from hydraulic fracturing stress measurements) is slightly compressive. Thus, even if we assume that the tensile strength is negligible, tensile fracturing is not expected to occur. Therefore additional sources of tensile stress, related to the drilling process itself, are needed to explain the formation of the
observed fractures. Other sources of tensile stress are (1) the thermal stress \( \sigma_T \) induced by cooling of the borehole wall due to circulation of relatively cold drilling mud, and (2) increased borehole pressure during drilling operations. Both additional stresses are uniform around the wellbore and therefore do not influence the orientation of the fracture, but they may lower the circumferential stress sufficiently to allow the initiation of a fracture. Thus the circumferential stress \( \sigma_{\theta \theta} \) acting on the borehole wall can be written as the sum of tectonic stress, thermally induced stress \( \sigma_T \), and mud pressure \( P_b \):

\[
\sigma_{\theta \theta} = S_H + S_h - 2(S_H - S_h) \cos(2\theta) - \Delta P + \sigma_T - 2P_b .
\]  

(5)

\( \Delta P \) is the difference between the pressure of the drilling mud \( P_m \) and the pressure of the porefluid \( P_p \). \( \theta \) is the angle with respect to the orientation of \( S_H \). The pressure in the borehole \( P_b \) is calculated as the sum of the hydrostatic pressure \( P_{\text{slug}} \) of the mud and 95% of the pumping pressure at the surface \( P_{\text{surf}} \) leading to

\[
P_b = P_{\text{slug}} + 0.95P_{\text{surf}} .
\]

This calculation of the downhole mud pressure \( P_b \) assumes the very small pressure loss of only 5% which is justified by the fact that in the KTB well a thixotropic mud was used, which transmits the first pressure peak when the pumps start with very little pressure loss down the hole (B. Engeser, personal communication, 1995). When the mud starts to flow, higher values for the dynamic pressure losses have to be used, which leads to a lower downhole pressure \( P_b \). As we are only interested in the peak pressure the rock ever experienced during drilling, we use the pressure peaks at the start of the pumps with a pressure loss of only 5%.

The minimum tangential stress concentration is found at angles \( \theta \) of 0° and 180° and controls the occurrence of tensile fractures. In traditional hydraulic fracturing theory it is assumed that in order to initiate a fracture, the tensile strength of the rock (typically tensile strengths of rocks from KTB vary around 10 MPa [Röckel and Nitsau, 1989, 1992]) has to be overcome. This is because hydraulic fracturing experiments are usually placed in sections of the borehole not affected by natural fractures and because a large new fracture is propagated away from the wellbore. To the contrary, drilling-induced fractures are initiated preferably, and most easy in sections of the borehole where small flaws in the borehole wall can be used as starting points for the development of fractures because there no tensile strength has to be overcome. Thus we conclude that the tensile strength of the rock does not need to be considered for the analysis of drilling-induced fractures.

The influence of the pore pressure \( P_p \) can be neglected for the same reasoning as discussed above with the hydraulic fracturing experiments. Equation (5) is then written as

\[
\sigma_{\theta \theta} = 3S_h - S_H + \sigma_T - P_b \leq 0 ,
\]

which can be solved for \( S_H \)

\[
S_H = 3S_h + \sigma_T - P_b .
\]

To give the most conservative estimate, and as it is unknown how much cooling and/or pumping pressure was necessary to initiate the fractures, the maximum horizontal principal stress \( S_h \) is calculated for two different cases: (1) the maximum amount of thermally induced stress \( \sigma_T \) and the maximum available pumping pressure (95% of \( P_{\text{surf}} \)) plus the hydrostatic pressure of the mud \( P_{\text{slug}} \) are acting on the borehole wall (equation (6) with \( P_b = 0.95P_{\text{surf}} + P_{\text{slug}} \)), or (2) only the hydrostatic mud pressure \( P_{\text{slug}} \) is acting on the borehole wall (equation (7)).

\[
S_H = 3S_h + \sigma_T - P_b .
\]  

(6)

\[
S_H = 3S_h - P_{\text{slug}} .
\]  

(7)

Equations (6) and (7) lead to lower and upper bound estimates for the magnitude of \( S_H \), respectively.

In order to estimate the magnitude of \( S_h \), use equations (6) and (7) from the occurrence of drilling-induced tensile fractures, the magnitude of \( S_h \), the mud pressure during drilling \( P_{\text{slug}} \), and the thermally induced stress \( \sigma_T \), have to be known. As discussed above, the downhole pressure \( P_b \) is the sum of the hydrostatic pressure \( P_{\text{slug}} \) and 95% of the wellhead pressure \( P_{\text{surf}} \) which is continuously recorded at the surface. For this analysis, the maximum pressure the rock has experienced before the fracture is detected needs to be determined. For this purpose, it has to be taken into account that pressure peaks recorded when the borehole has reached a certain depth raise the pressure also in the entire mud column above this depth and can overcome the pressure recorded for these shallower depth sections at earlier times. The values for the maximum downhole pressure \( P_b \) range between 55 MPa and 80 MPa, which is about 20 MPa above the hydrostatic mud pressure at any given depth (Figure 7b).

The thermally induced stress is calculated, following Stephens and Voigt [1982] and Mool and Zoback [1990], by

\[
\sigma_T = -\frac{\alpha E \Delta T}{1 - \nu} ,
\]  

(8)

where \( \alpha \) is the thermal expansion coefficient, \( E \) is the Young's modulus, \( \nu \) is the Poisson's ratio, and \( \Delta T \) is the temperature difference between the undisturbed rock temperature \( T_r \) and the mud temperature \( T_m \). The acquisition of appropriate values for all these parameters is described in Appendix B. The induced thermal stress ranges between -20 MPa and -80 MPa (Figures 7a).

Both methods for the estimation of stress magnitudes described above assume that the vertical stress is a principal stress. The validity of this assumption is discussed in detail in Appendix C using information from the occurrence of drilling-induced fractures and their orientation with respect to the borehole axis.

**Orientation of \( S_H \) From Analysis of Breakouts and Drilling-Induced Tensile Fractures**

Three methods were used to establish a continuous stress orientation profile in the KTB main hole: (1) the analysis of breakouts using four-arm caliper data recorded by the borehole geometry tool (BGT) and FMS, (2) the identification of breakouts in BHTV images, and (3) the investigation of drilling-induced tensile fractures using FMS/FMI measurements. The mean orientation of the greatest horizontal principal stress \( S_h \) was calculated for depth intervals of 50 m for
The average stress orientation between N150°E and N170°E is consistent with other determinations of stress orientation in the area and with the general stress orientation of N145°±26°E in central Europe described by Müller et al. [1992] and demonstrated by a map of the area with the orientations of $S_h$ marked by arrows (Figure 1). In the KTB pilot hole, the orientation of the principal horizontal stresses was determined by log-based methods like the analysis of well bore breakouts [Mastin et al., 1991] and drilling-induced tensile fractures as well as by core-based methods like differential strain analysis [Baumann, 1993], anelastic strain recovery [Zang et al., 1989], wave velocity analysis [Zang et al., 1990], and analysis of centerline fractures and core disking structures [Röckel et al., 1992; Wolter et al., 1990]. The results of the above methods are generally quite consistent with each other and with the regional orientation of $S_h$ in central Europe of N145°±26° determined by Müller et al. [1992] (Figure 1).

### Calibration of ISMS in the Pilot Hole

In order to apply the methods described above for the determination of the magnitudes of the greatest horizontal principal stresses, $S_h$ at great depth in the KTB main hole, it is essential first to calibrate these techniques where control on the magnitudes of the principal stresses is available. The KTB pilot hole is an ideal borehole for this purpose as (1) the magnitudes of $S_h$ and $S_v$ were determined by hydraulic fracturing experiments, (2) breakouts and drilling-induced fractures were observed by high-quality BHTV and FMS measurements, and (3) the lithology is identical to the KTB main hole as the same rock units are stacked up by ancient thrust faults [Duyser et al., 1994]. The similarity of the lithologies in the two boreholes allows us to use the same rock types for the calibration of the methods as for the application in the KTB main hole, which is a...
Figure 8. (a) Using the most reliable stress orientation data from any depth, a profile of the orientation of \( S_n \) in the depth section 3.2 km to 8.6 km is derived. For this profile, breakout orientations derived from BHTV measurements were used between 3200 m and 6800 m; below this depth the breakout analysis of four-arm caliper data recorded by the borehole geometry tool (BGT) and the FMS is used. The stress orientations derived from the different logs are averaged over depth sections of 50 m. Except an abrupt change in stress orientation at about 7200 m, the orientation of \( S_n \) is quite constant over the entire depth interval. (b) Standard deviation and the reciprocal number of measurements are given for each 50 m depth interval of Figure 8a. High values of both standard deviation and reciprocal number of measurements indicate a badly constrained stress orientation and vice versa.

great advantage as different rock types commonly react mechanically quite differently.

The occurrence of breakouts in the KTB pilot hole is described by Mastin et al. [1991]. Analysis of BHTV images of the breakouts occurring in the amphibolites below 3500 m depth in the KTB pilot hole allows us to determine a mean breakout opening angle of 40° at about 3590 m and 3775 m depth. With the effective in situ rock strength \( C_{\text{eff}} \) for the respective depths (Figure 5), we determined a lower and upper bound for the magnitude of \( S_n \), given in Figure 9 by open and solid circles, respectively. We restricted the calibration to the amphibolite section below 3500 m depth, as also in the KTB main hole, we restricted the use of the breakout method to amphibolite sections in order to avoid influences of the strength anisotropy of the gneisses. For the magnitude estimation from breakouts, we used a very wide range of rock strengths (Figure 5) in order to give a conservative stress estimation; thus the range of \( S_n \) magnitudes is quite large, but it is definitely in agreement with the linear extrapolation of the \( S_n \) magnitude derived from

Figure 9. Calibration of stress estimation methods. \( S_n \) and \( S_h \) magnitudes determined from hydraulic fracturing (HF) are displayed by solid and open squares, respectively. Error bars indicate the uncertainty in the determination of the \( S_n \) magnitude by hydraulic fracturing. The magnitudes of \( S_n \) estimated by the methods described above are given by the shaded area (drilling-induced fractures) and by open and solid circles (breakouts, BO), respectively. The results of the two methods are in very good agreement with the stress magnitudes derived from hydraulic fracturing tests to 3 km depth (solid and open squares). This demonstrates that the two estimation methods are capable of revealing reliable estimates of the magnitude of \( S_n \).
hydraulic fracturing experiments (open squares, Figure 9) and with the estimation of the $S_h$ magnitude by the analysis of drilling-induced fractures as discussed below.

Drilling-induced fractures in the KTB pilot hole were analyzed from FMS measurements below a depth of 3000 m [Brudy, 1995]. They were found to be ubiquitous in the entire investigated depth section from 3 km to 4 km depth. Using the appropriate values for thermal stress $\sigma_t$ and maximum downhole pressure $P_d$ in the KTB pilot hole, we estimated a lower and upper bound for the $S_h$ magnitude applying equations (6) and (7). The resulting range of $S_h$ magnitudes accounting for initiation of drilling-induced fractures is given in Figure 9 by the shaded area. The estimated magnitude range coincides very well with the hydraulic fracturing results at 3 km depth, with the linear extrapolation of the hydraulic fracturing results to depths below 3 km, and with the magnitude estimations from breakouts at about 3590 m and 3775 m depth.

These results in the KTB pilot hole demonstrate that both methods are capable of revealing reliable estimates of the magnitude of $S_h$ and can be used for the estimation of $S_h$ at great depth in the KTB main hole where conventional hydraulic fracturing is not possible.

Estimation of the Magnitude of $S_h$ at Great Depth

The presence of both breakouts and drilling-induced fractures at the same depths in the KTB main hole allows us to utilize both methods for estimating the magnitude of $S_h$. Using the combination of the two methods, not only the magnitude of $S_h$ for a known magnitude of $S_v$ can be estimated, but combinations of $S_h$ and $S_v$ values can be determined which can explain both the occurrence of breakouts and the occurrence of drilling-induced fractures. An additional constraint on the magnitude of $S_h$ is provided by the hydraulic fracturing tests at 6 km and 9 km depth which give a lower bound for the value of $S_v$.

To find the possible combinations of $S_h$ and $S_v$, we represent the two estimates in a plot of $S_v$ versus $S_h$. This was done at 200-m intervals from 3.2 km depth to 6.8 km. Four examples of these plots are shown in Figure 10 for the depths 4 km, 5 km, 6 km, and 6.8 km. For arbitrary $S_v$ values, the respective $S_h$ values are calculated using equation (4) for the breakout analysis equations (6) and (7) for the analysis of drilling-induced fractures. From Figure 5, breakout opening angle and in situ rock strength were derived as functions of depth. Using the respective breakout opening angle and the upper and lower bound in situ rock strength, $C_p$ for each depth an upper and lower bound for the magnitude combinations can be determined (equation (4)). Upper and lower bounds from the breakout analysis are marked in Figure 10a by a and b, respectively. The very large range of uncertainty in rock strength (Figure 5) we are using insures that the estimation of the $S_h$ magnitude by borehole breakouts is conservative.

Upper and lower bound estimations of the magnitude combinations from the analysis of drilling-induced fractures are marked in Figure 10a by c and d, respectively. These bounds

![Figure 10](image-url)
a respective range of magnitudes of $S_n$ which explain the occurrence of breakouts and drilling-induced fractures.

In Figure 11, all knowledge on the stress magnitudes is combined in one profile. The vertical stress $S_v$ is the overburden. The $S_h$ and $S_n$ magnitudes determined by hydraulic fracturing in the KTB pilot hole to a depth of 3 km are represented by solid and open squares, respectively. Uncertainties for the $S_n$ magnitude are displayed in Figure 2. The lower bound magnitudes of $S_n$ derived from the modified hydraulic fracturing experiments at 6 km and 9 km depth are given by solid diamonds. Between 3 km and 6.8 km the results of the combined analysis of breakouts and drilling-induced fractures are shown. At each depth, three $S_n$ magnitudes and the respective ranges of $S_n$ magnitudes are presented. From Figure 10 it is obvious that for the least and intermediate $S_n$ magnitude (triangles and crosses) a range of $S_n$ values results, whereas for the greatest $S_n$ magnitude (diamonds) only a single $S_n$ magnitude is possible. Below 6.8 km no BHTV data, and thus no information on the breakout opening angle, were available. Therefore, below this depth only the drilling-induced fractures found at 7 km and at 7.7 km depth can be used to estimate a range of $S_n$ magnitudes. In this depth section, as we have to rely only on the estimation from drilling-induced fractures, we can constrain the $S_n$ range only between a lower bound given by the interpolation of the hydraulic fracturing results at 6 km and 9 km depth and an upper bound given by the overburden load. Assuming the lower bound value for the magnitude of $S_n$ (interpolation between the lower bounds established by hydraulic fracturing tests at 6 km and 9 km depth), the possible magnitude range for $S_n$ is given by solid triangles at 7 km and at 7.7 km depth. Assuming a $S_n$ magnitude equal to the vertical stress results in a range of $S_n$ magnitudes between 503 MPa and 544 MPa. As the combined analysis constrains the $S_n$ magnitude at 6.8 km to values very close to the lower bound given by the hydraulic fracturing tests, we consider it unreasonable that the $S_n$ magnitude is close to the vertical stress at 7 km and at 7.7 km depth. Therefore we did not include the upper bound estimate for these depths in Figure 11 (Figure 13 and the related discussion show that also the assumption of an upper bound $S_n$ magnitude at 7 km and 7.7 km depth is not changing the principal conclusion, see below).

In summary, the profile of the stress magnitudes demonstrates that (1) below 1 km depth the relation of the stress magnitudes ($S_h < S_v < S_n$) indicates a strike-slip tectonic regime and (2) that the differential stress $(S_v - S_n)$ is increasing with depth at least to 7.7 km depth. The redundancy of the ISMS by using both breakouts and drilling-induced fractures for the estimation of the stress magnitudes is of great importance as it allows us to incorporate extremely large uncertainties for the rock strengths in the breakout analysis (Figure 5) and the uncertainty about the influence of thermal stress and pumping pressure on the initiation of drilling-induced fractures (see discussion of equations (6) and (7)), but still reasonably narrow ranges for the $S_h$ and $S_n$ magnitudes are the result of the stress estimation.

To investigate the hypothesis that the brittle crust is in frictional equilibrium controlled by the strength of preexisting optimally oriented faults, we plotted Mohr diagrams [Mohr, 1914; Jaeger and Cook, 1979] for the results of the combined analysis at 4 km, 5 km, 6 km, and 6.8 km depth (Figure 12). At each depth five Mohr circles for the following combinations of $S_h$ and $S_n$ magnitudes are displayed: the least $S_h$ value with respective least and greatest $S_n$ value, an intermediate $S_h$ value.
with respective least and greatest $S_u$ value, and the greatest $S_u$ value with the respective $S_u$ value. In Figure 12e, four Mohr circles for the stress estimation at 7.7 km depth are presented. For these Mohr circles, lower and upper bound ($S_u$=$S_l$) estimates of the $S_u$ magnitude and the respective upper and lower bounds for $S_u$ are used. In all Mohr diagrams (Figure 12), failure lines for a coefficient of friction of 0.8 and 0.6 are included. Without exception, all Mohr circles reach or overcome the failure line for a coefficient of friction of 0.6. Even the assumption of a $S_u$ magnitude equal to $S_u$ at 7.7 km (Figure 12e) leads to a stress state which is capable of inducing slip on favorably oriented faults. All this leads us to the conclusion that the hypothesis of a frictional equilibrium at preexisting faults is correct at the KTB site to at least a depth of 7.7 km.

A summary of the differential stress $S_l$-$S_u$ determined at KTB is given in Figure 13 in comparison to a theoretically calculated profile. The strength profile in the upper crust is calculated for the case of a strike-slip stress regime, a coefficient of friction $\mu$ between 0.6 and 0.7 and hydrostatic pore pressure (all these assumptions are confirmed for the KTB site by stress and pore pressure measurements).

To compute the theoretical stress difference in a strike-slip stress regime where $S_l$ and $S_u$ are equal to $S_u$ and $S_l$, respectively, we extrapolated the measurements of $S_u$ and determined the value of $S_u$ required to cause frictional faulting in this tectonic regime.

In order to assess the entire crustal strength, we calculated the ductile creep strength for lower crustal rocks with very low and very high strength, using the creep properties of Adirondack and Pikipiton Granulite as examples. To calculate such a strength profile, the temperature profile and the strain rate must be known. Temperatures in the lower crust and upper
shield areas, where the upper mantle due to relatively low temperatures is assumed to have appreciable strength. No lithospheric deformation is expected to occur in such regions, as the upper mantle is capable of supporting much of the total force available.

Conclusions

Analysis of BHTV and FMS/FMI images of the pilot and main borehole for occurrence and orientation of breakouts and drilling-induced tensile wall fractures permits determination of the orientation of the principal stresses to a depth of 8.6 km. The applied method intrinsically assumes that the vertical stress is a principal stress. The general validity of this assumption is strongly supported by the spatially very restricted occurrence of drilling-induced fractures, which are inclined to the well bore axis and thus to the vertical direction. The observed orientation of the greatest horizontal principal stress $S_h$ is remarkably uniform (N150°E-N170°E) along the entire investigated interval from 3.2 km to 8.6 km depth. Besides ubiquitous small-scale variations of the stress orientation, the only significant deviation from the overall trend is observed below 7.2 km depth, where the lower bound of the intersection of the borehole with a major fault zone, the SE1 reflector, is located.

Hydraulic fracturing experiments carried out in the KTB pilot hole reveal values for the magnitude of both horizontal principal stresses to a depth of 3 km. Below, lower bound values for the magnitude of the least horizontal principal stress $S_3$ are found by modified hydraulic fracturing experiments at 6 km and 9 km depth in the KTB main hole. Two methods are derived to estimate the magnitude of the greatest horizontal principal stress $S_h$ from the occurrence of breakouts and of drilling-induced fractures. Both methods are calibrated in the KTB pilot hole where independent control of the magnitude of the horizontal principal stresses is given by hydraulic fracturing measurements and are found to be capable of deriving reliable estimates of the greatest horizontal principal stress $S_h$ for the case that the least horizontal principal stress $S_3$ is known.

The magnitude of $S_h$ for the depth section between 3.2 km and 6.8 km depth was estimated by the combined analysis of breakouts and drilling-induced fractures. Between 6.8 km and 7.7 km, both failure types do occur but because of the lack of BHTV measurements, only drilling-induced fractures observed around 7 km and around 7.7 km could be used to estimate the magnitude of $S_h$ at this depth range.

The resulting state of stress at 7.7 km depth allows slip on favorably oriented faults with coefficients of friction between 0.6 and 0.7, values consistent with laboratory measurements of friction. These findings demonstrate the validity of the hypothesis that the state of stress in the brittle crust is limited by the frictional equilibrium on preexisting favorably oriented faults with realistic coefficients of friction consistent with laboratory measurements. The presented stress profile is the deepest justification of this concept which is commonly assumed for stress profiles throughout the brittle part of the crust.

The results also indicate that the upper, brittle part of the crust in areas of moderately high heat flow as central Europe is capable of supporting forces equivalent in magnitude to plate driving forces [Zoback et al., 1993]. Thus, in such areas, where the strength of the lower crust and the upper mantle is assumed to be low the brittle crust is expected to act as a "stress guide".
Figure A1. (a) Polyaxial strength measurements of Solenhofen limestone [Mogi, 1971] plotted on curves showing the state of stress at failure for a coefficient of friction \( \mu = 0.4 \) calculated with the effective shear strain energy criterion. All stresses are normalized by the uniaxial compressive rock strength \( C_0 \). (b) Polyaxial strength measurements of Dunham dolomite [Mogi, 1971] plotted on curves showing the state of stress at failure for a coefficient of friction \( \mu = 0.7 \). All stresses are normalized by the uniaxial compressive rock strength \( C_0 \). Different symbols indicate various values of \( S_2/C_0 \) as explained in the lower right corner.

Appendix A: Effective Shear Strain Energy Failure (ESEF) Criterion

Commonly used rock strength criteria take into account only variations in the maximum differential stress and neglect the influence of the intermediate principal stress. True polyaxial strength measurements by Mogi [1971] show that the intermediate principal stress can significantly influence rock strength. As in situ rock strength is an input parameter in the breakout analysis and because the state of stress at the borehole wall is polyaxial (\( \sigma_1 > \sigma_2 > \sigma_3 \)), we decided to use a strength criterion which includes the influence of the intermediate stress following Vernik and Zoback [1992]. The effective shear strain energy failure criterion (ESEF criterion), developed by Wiebold and Cook [1968], allows us to calculate polyaxial states of stress at the point of failure using triaxial rock strength measurements.

The basic prerequisite for the ESEF criterion is a homogeneous and isotropic microcrack population in the rock. When the rock is under compression, strain energy is stored in the rock mass. This energy consists of two parts, the elastic deformation energy and an additional energy due to the movement of crack surfaces in the applied stress field. The magnitude of the latter energy depends on the effective shear stress \( \tau_{eff} = \mu \sigma_{13} \) on the crack planes and therefore on the orientation of the cracks. Here \( \tau_{eff} \) is the difference between the shear stress \( \tau \) on a plane induced by the respective state of stress and the frictional strength of the plane (\( \mu \), coefficient of friction; \( \sigma_{13} \), normal stress on plane). Thus the so-called effective shear strain energy \( W_\tau \) is stored only at cracks which are oriented relative to the stress field in a way that slip on the crack can occur. This is only the case if \( \tau_{eff} \) is positive.

According to the ESEF criterion, failure occurs when the effective strain energy \( W \), the sum of \( W_\tau \) over all cracks which
are able to move in the applied stress field, reaches a critical value \( W_{\text{cr}} \). Knowing the coefficient of friction \( \mu \) the critical energy \( W_{\text{cr}} \) can be calculated from the results of an uniaxial strength test.

Polyaxial strength investigations are rarely carried out due to the great experimental difficulties. Therefore the applicability of the ESEF criterion was demonstrated only by very few polyaxial strength data from Karroo Dolerite and from Bowral Trachyte [Hoskins, 1967].

Polyaxial strength data for Dunham Dolomite and Solenhofen Limestone [Mogi, 1971] give further evidence for the applicability of the ESEF criterion (Figure A1). The stresses at failure are plotted in a diagram which shows stress states of constant effective shear strain energy \( W_e \) calculated for the coefficients of friction of \( \mu=0.7 \) and \( \mu=0.4 \). Both data sets fit very well to the diagrams and support the applicability of the ESEF criterion to isotropic rock.

Appendix B: Analysis of Thermal Stress Affecting Drilling-Induced Tensile Wall Fractures

The borehole wall and the surrounding rock mass in the lower part of the borehole are cooled by the drilling mud, which is pumped into the borehole with surface temperature. On the way down to the drill bit, the temperature of the mud increases but does not reach the undisturbed temperature of the rock and is therefore capable of extracting heat from the rock. The radially varying temperature field around the borehole caused by this effect depends on the thermal diffusivity of the rock, the radius of the borehole, and the time since cooling started. Integrable expressions for the temperature field around a cylindrical borehole in an infinite medium are given by Ritchie and Sakakura [1956] for times of \(-1\) day through infinity. The stresses resulting from a radially varying temperature field are derived by Nowacki [1962] and specified for the case of a cylindrical borehole by Stephens and Voight [1982]. For the consideration of fracture initiation, only the stress directly at the borehole wall is of importance and can be calculated by the simple expression

\[
\sigma_T = -\frac{\alpha E \Delta T}{1 - \nu} . \tag{B1}
\]

The calculation of the thermal stress according to equation (B1) involves determining the linear coefficient of thermal expansion \( \alpha \), Young's modulus \( E \), Poisson's ratio \( \nu \), and the temperature difference \( \Delta T \) between the undisturbed temperature of the rock and the temperature after circulation of the drilling mud.

The coefficient of thermal expansion of a rock is best represented by the average of the coefficients of thermal expansion of the minerals constituting the rock [Skinner, 1966]. The abundances of quartz, mica, amphibolite, garnet, chloride, biotite, muscovite, and plagioclase are analysed routinely in the KTB holes and offer the possibility to calculate the coefficient of thermal expansion from the mineral composition. Except quartz (1.0x10^{-5} °K^{-1}), all these minerals have a very similar coefficient of thermal expansion (about 0.6x10^{-5} °K^{-1}), which allows calculation of the linear coefficient of thermal expansion for the rock by

\[
\alpha = \sum_i \alpha_i q_i + (1 - \sum_i q_i) \alpha_{\text{others}} . \tag{B2}
\]

Figure B1. The linear coefficient of thermal expansion \( \alpha \) of the rock matrix is calculated according to equation (B2) from the thermal expansion coefficient of the main mineral constituents. Except for quartz (1.0x10^{-5} °K^{-1}), all other minerals have very similar coefficients of thermal expansion (0.6x10^{-5} °K^{-1}). The curve is smoothed by a median filter.

The resulting coefficients of thermal expansion (Figure B1) may differ to some minor extent from the actual in situ coefficient of thermal expansion of the rock. The reason for this is twofold, first, the volumetric expansion is not only caused by the expansion of the minerals but also by the creation of new porosity due to different expansion coefficients of the various mineral grains, and second, the values for the expansion coefficients are determined for constant pressure and therefore do not reflect the dependence of thermal expansion on the pressure.

The elastic parameters Young's modulus \( E \) and Poisson's ratio \( \nu \) are derived from \( P \) and \( S \) wave sonic velocities measured in the direction parallel to the borehole axis as an average value for all azimuths around the well bore. Ideally, in this analysis, the static moduli as a function of azimuth and depth should be used; however, no such data are available. The resulting Young's modulus and Poisson's ratio are plotted in Figure B2.

The undisturbed temperature of the rock is derived from temperature equilibration measurements which lead to a temperature gradient of 27.4K/km [Zoth, 1993; Clauser et al., this issue]. To find the amount of drilling-induced cooling of the rock, temperature logs carried out during measurement phases and recordings of the downhole temperature during drilling were compared to the equilibrium temperature profile. As the temperature logs were carried out several hours after circulation was stopped, the temperature found in these logs is always higher than the temperature recorded by the
measurements carried out during drilling. This is due to the fact that the steep temperature gradient in radial direction around the well bore, produced by the drilling process, allows the drilling mud and the borehole wall to significantly heat up in short time periods after the end of circulation. As we need to use the maximum amount of cooling the rock experienced during drilling, we choose to use the difference between the measurements while drilling and the equilibrium profile (Figure B3).

Appendix C: Is One Principal Stress Vertical?

It is very common to assume that one principal stress is vertical and its magnitude is the overburden load $5 \cdot \rho g z$, where $\rho$ is the average density of the overburden, $g$ is the acceleration of gravity, and $z$ is the depth. The validity of this assumption was assessed by McGarr and Gay [1978], who concluded that it is basically valid, but slight departures from this rule are common. Most of the data they used for this study were obtained in mines, "often in regions of complex geology", and thus they expected that orientations of stress in other areas might conform more closely to this assumption. In their compilations of different types of in situ stress indicators in of North America, Zoback and Zoback [1980, 1991] made similar arguments, which were later extended to the global compilation of in situ stress indicators [Zoback et al., 1989; Zoback, 1992]. Further indications for this assumption to be valid are given by the analysis of fault slip data [Angelier, 1984] and focal plane solutions [Gephart and Forsyth, 1984] at various sites which show that one of the principal stresses is often within a few degrees of vertical.

Analyses of drilling-induced fractures in boreholes where no principal stress is parallel to the borehole axis [Brudy and Zoback, 1993, Peska and Zoback, 1995] demonstrate that in such a situation drilling-induced fractures are not aligned with
the borehole axis. The amount of deviation is dependent on the stress regime and on the orientation of the borehole with respect to the principal stresses [Peksa and Zoback, 1995].

The KTB main hole is drilled straight vertical (less than 1° deviation) to a depth of more than 7 km, and the KTB pilot hole is only deviated slightly from vertical. Therefore the orientation of drilling-induced fractures in the KTB boreholes can be used to assess the question of the vertical stress being a principal stress. If the vertical stress is a principal stress, drilling-induced fractures in these boreholes should preferably be aligned with the borehole axis [Brudy and Zoback, 1993; Peksa and Zoback, 1995]. In the KTB pilot hole, deviated fractures are observed quite infrequently [Brudy, 1995], and in the KTB main hole, deviated drilling-induced fractures are even rarer. Further indication for the vertical stress being a principal stress is given by the fact that most of the deviated drilling-induced fractures seem to be related to local perturbations of the stress field caused by faults cutting the borehole. Local perturbations of the stress field associated with faults were observed by Shamir and Zoback [1992] in the Cajon Pass borehole and by Barton and Zoback [1994] in the KTB main hole and several other boreholes. Small-scale perturbations of the breakout orientation observed in the KTB main hole [Brudy et al., 1995] are further indication for stress perturbations associated with faults crosscutting the borehole. All these authors relate the local stress perturbations at faults to slip along the faults possibly initiated by the drilling process. This means that most of the deviated drilling-induced fractures seem to be caused by locally perturbed stress states close to faults where the vertical stress is not a principal stress. In borehole sections where the stress field is not perturbed by slip on faults, the observed drilling-induced fractures are parallel to the borehole axis and thus indicate that generally, the vertical stress at the KTB site is a principal stress.

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